D1(H) Math paper-1st Gr-A (ii) Ex-8

$$C = \frac{1}{2}\log\{1 + 2c\cos\alpha + \dots\}$$

①

$\cos\alpha\cos\alpha - \frac{1}{2}\cos^2\alpha\cos2\alpha + \frac{1}{3}\cos^3\alpha\cos3\alpha - \dots \infty$

$\cos\alpha\sin\alpha - \frac{1}{2}\cos^2\alpha\sin2\alpha + \frac{1}{3}\cos^3\alpha\sin3\alpha - \dots$

Let $C = \cos\alpha\cos\alpha - \frac{1}{2}\cos^2\alpha\cos2\alpha + \frac{1}{3}\cos^3\alpha\cos3\alpha \dots$

$S = \cos\alpha\sin\alpha - \frac{1}{2}\cos^2\alpha\sin2\alpha + \frac{1}{3}\cos^3\alpha\sin3\alpha - \dots$

$$\therefore C + iS = (\cos\alpha)(\cos\alpha + i\sin\alpha) - \frac{1}{2}\cos^2\alpha(\cos2\alpha + i\sin2\alpha) + \frac{1}{3}\cos^3\alpha(\cos3\alpha + i\sin3\alpha) - \dots$$

$$= \cos\alpha\, e^{i\alpha} - \frac{1}{2}\cos^2\alpha\, e^{i2\alpha} + \frac{1}{3}\cos^3\alpha\, e^{3i\alpha} - \dots$$

Let $x = \cos\alpha . e^{i\alpha}$

$$\therefore C + iS = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \dots$$

$$= \log(1+x) = \log(1 + \cos\alpha . e^{i\alpha})$$

$$= \log\{1 + \cos\alpha(\cos\alpha + i\sin\alpha)\}$$

$$= \log\{1 + \cos^2\alpha + i\sin\alpha\cos\alpha\}$$

$$= \frac{1}{2}\log\{(1+\cos^2\alpha)^2 + \sin^2\alpha\cos^2\alpha\} + i\tan^{-1}\frac{\sin\alpha\cos\alpha}{1+\cos^2\alpha}$$

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(2)

$$\therefore C+iS = \frac{1}{2}\log\left\{1+2\cos^2\alpha+\cos^4\alpha+(1-\cos^2\alpha)\cos^2\alpha\right\} + i\tan^{-1}\frac{\sin\alpha\cos\alpha}{1+\cos^2\alpha}$$

Equating

$$C = \frac{1}{2}\log(1+3\cos^2\alpha) \text{ Ans}$$

$$S = \tan^{-1}\frac{\sin\alpha\cos\alpha}{1+\cos^2\alpha} \text{ Ans}$$

1 (iv) ऊपर के तरह बनेगा।

1.(v.) 1(ii.) के तरह बनेगा

1.(vi) इसी तरह

1.(vii) उसी तरह बनेगा।

$$\log(\alpha+i\beta) = \frac{1}{2}\log(\alpha^2+\beta^2) + i\tan^{-1}\beta/\alpha$$

1.(viii) $C = \cos\frac{\pi}{3} + \frac{1}{2}\cos\frac{2\pi}{3} + \frac{1}{3}\cos\frac{3\pi}{3} + \cdots$

$$S = \sin\pi/3 + \frac{1}{2}\sin\frac{2\pi}{3} + \frac{1}{3}\sin\frac{3\pi}{3} + \cdots$$

$$C+iS = (\cos\pi/3 + i\sin\pi/3) + \frac{1}{2}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) + \cdots$$

$$= e^{i\pi/3} + \frac{1}{2}e^{i2\pi/3} + \frac{1}{3}e^{i3\pi/3} + \cdots$$

let $e^{i\pi/3} = x$

$$\therefore C+iS = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots$$

$$= -\log(1-x) = -\log\left\{1-e^{i\pi/3}\right\}$$

$$= -\log\left\{1-(\cos\pi/3 + i\sin\pi/3)\right\}$$

$$= -\log\left\{1-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right\} \quad \left(\begin{array}{l}\cos\pi/3 = \cos 60 = \frac{1}{2} \\ \sin\pi/3 = \sin 60 = \sqrt{3}/2\end{array}\right)$$

$$= -\log\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)$$

$$= -\left[\frac{1}{2}\log\left\{\left(\frac{1}{4}+\frac{3}{4}\right)\right\} - i\tan^{-1}\frac{\sqrt{3}/2}{1/2}\right]$$

$$= -\left[\frac{1}{2}\log\frac{4}{4} - i\tan^{-1}\sqrt{3}\right]$$

$\therefore z = \frac{1}{2}\log 1 + i\tan^{-1}\sqrt{3} = 0 + i\,\pi/3$

$\therefore c = 0,\ s = \pi/3$ Ans